Spike train analysis and Gibbs distributions

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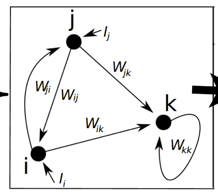
Introduction

Spikes in sensory neurons are conveyed collectively to the cortex using correlated binary patterns (in space and time) which constitute "the neural code". Since patterns occur irregularly it is appropriate to characterize them using probabilistic descriptions or statistical models. Two major approaches attempt to characterize the spike train statistics: The Maximum Entropy Principle (MaxEnt) and Neuronal Network modeling (N.N). Remarkably, both approaches are related via the concept of Gibbs distributions. MaxEnt models are restricted to time-invariant Gibbs distributions, via the underlying assumption of stationarity, but this concept extends to non-stationary statistics (not defined via entropy), allowing to handle as well statistics of N.N models and GLM with non-stationary dynamics. We show in this poster that, stationary N.N, GLM models and MaxEnt models are equivalent via an explicit mapping. This allows us, in particular, to interpret the so-called "effective interactions" of MaxEnt models in terms of "real connections" models.

Gibbs from N.N. models and MaxEnt

N.N Models: From neuronal network models to Gibbs potentials [2].

- **LIF**
- **GLM**



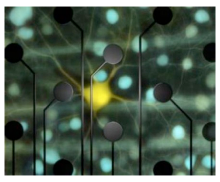
Conditional probabilities

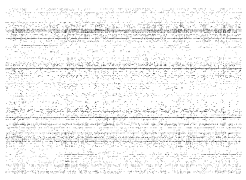
Parameters

$$\phi(\omega) = \log \mathbb{P}[\omega(0) \mid \omega_{-\infty}^{-1}]$$

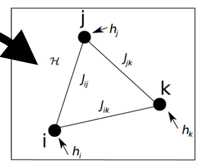
The Gibbs potential is an explicit non linear function of synaptic weights and stimulus

MaxEnt: From experimental data to Gibbs potentials [1].





$$\mathcal{H}(\omega) = \sum_{l=1}^{2^{NL}} h_l \mathcal{O}_l$$



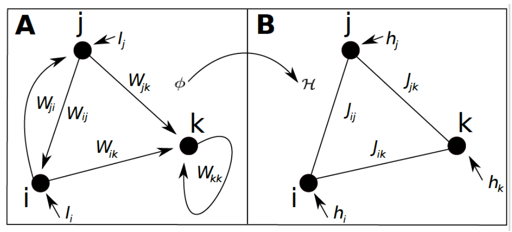
Lagrange multipliers h_l have no interpretation in terms of the biological network.

Mapping

The potential $\mathcal{H}$ (Max Ent) and ϕ (N.N) are mapped onto each others via cohomology. Two cohomologous potentials correspond to the same Gibbs distribution (spatio-temporal spikes patterns have the same probabilities in the two models) if they satisfy the following equation.

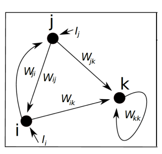
Cohomology equation: Two Gibbs potentials are cohomologous if there exist a function f and a constant K such that:

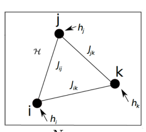
$$\mathcal{H}(\omega_0^D) = \phi(\omega_0^D) - f(\omega_0^{D-1}) + f(\omega_1^D) + K$$



Dimensions

The Gibbs potential obtained from N.N models can be expressed as a sum of spike interactions. The classical summation of spike interactions, on which MaxEnt model is grounded, leads to a plethora of redundant terms and a huge and artificial spread of dimensionality. The pairwise interactions are not easily related to synaptic interactions, they also depend on the stimulus as shown by Cocco and Monasson [3] for the Ising spatial model (no memory in dynamics). We extend it here to the spatio-temporal case.





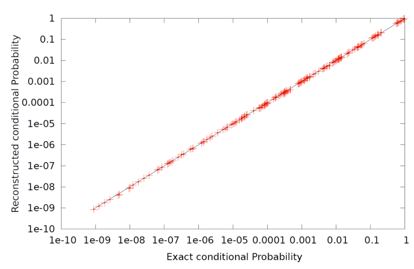
$$\phi(\omega) = \log \mathbb{P}[\omega(0) \mid \omega_{-D}^{-1}]$$

$$\sum_{i=1}^{N} h_i\omega_i + \sum_{i,j=1}^{N} J_{ij}\omega_i\omega_j + \ldots\ldots + \sum_{i,j=1}^{N} J^1_{ij}\omega_i\omega^1_j + \ldots + \sum_{i,j,k=1}^{N} \gamma^1_{ijk}\omega_i\omega^1_j\omega^1_k$$

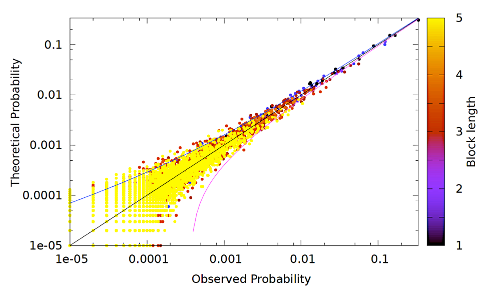
$N^2 + N$ $\qquad$ $2^{ND} - 2^{N(D-1)}$

# of parameters

Example



Exact conditional probabilities for blocks of range R obtained from the normalized potential ϕ , v/s exact conditional probabilities associated with the potential $\mathcal{H}$.



Empirical probabilities of blocks (darker lower length), obtained from a discrete leaky integrate and fire spike train of size T = 10^5 v/s the probabilities of the same blocks predicted by the Gibbs distribution with potential $\mathcal{H}$.

Perspectives

The Gibbs theory provides a very general and flexible way to build statistical models from data and N.N models and is suitable to model biological phenomena. This approach offers interesting perspectives, for example how spatio-temporal correlations are modified by adding a non stationary stimulus using the linear response. Other examples include Hidden Gibbs models (related to Binning [4]), Conditioning upon the future (related to the notion of anticipation) etc..

References

[1] J. C. Vasquez, O. Marre, A. G. Palacios, M. J Berry, and B. Cessac. J. Physiol. Paris, 106(3-4):120–127, 2012.

[2] S. Cocco, S. Leibler, and R. Monasson. PNAS, 106(33):14058–14062, 2009.

[3] R. Cofré and B. Cessac. Physical Review E, 107(5): 368–368, 2014.

[4] B. Cessac, A. Le Ny, Eva Löcherbach. Neural Computation, (In press), (2016).